cross thermal resistance is obtained (for instance, using thin graphited paper), by following the paint melting in time, more information about the distribution of the incident power can be obtained.

The following experiments on some well-known structures have been performed. The first experiment refers to two guides of rectangular and circular cross section, respectively, in which only the fundamental mode can propagate (Figs. 1 and 2).

For the second experiment we have realized a multimode guide, joined asymmetrically to an exciting standard rectangular guide, which can be rotated around its axis, in order to obtain a variable polarization of the impressed field. Thereafter, the dissipative sheet has been placed on the terminal section and the dissipation has been visualized for many polarizations of the exciting field.

Many configurations of the kind shown in Fig. 3, have been obtained showing that more than one mode is propagating.

Finally, in order to obtain the propagation of the TE_{01} mode only, using a well-known technique [3], the metallic continuous wall has been replaced by a wall made by an isolated copper wire tightly wound.

The results of Fig. 4 were obtained by performing the experiment with such a guide.

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A Note on "Submillimeter Wave Harmonic Mixing"

In their recent correspondence, Strauch et al. [1], described two ways to prove the generation of harmonics by a crystal harmonic generator. In their experiments they used two millimeter-wave klystrons, one of them was swept with Δf, the other acted as local oscillator operated in CW. The two outputs were mixed at the diode of a crossed-waveguide device. There, a fundamental or a harmonic mixing, respectively, took place. The difference frequency signals produced in this way were amplified in a 30 MHz IF amplifier. The detected video output was displayed on an oscilloscope. They observed upper and lower sidebands. The distances between these were 60, 30, 20, 15 · · · MHz, in general 60 MHz/n. Now they stated that all these beats were produced by harmonics, moreover they claimed to have observed more than 20 harmonics from a 72.9 GHz klystron. We have strong objections to these statements. This also holds for the papers of Murai [2], [3], who claimed to have observed harmonics of the order of 14 in a similar experiment.

If the rectified current of the diode contains harmonics of the frequency nf_1-mf_2, then frequencies of $(n+m)f_1$ should also be present with the same order of magnitude. With $n=m=20$ this means that the 40th harmonic would have to be generated in the diode with about the same intensity as the 20th harmonic IF. Besides the fact that this is very unlikely, we can give a more probable explanation of such beats as a result of our experiments. The fundamental beat frequency f_1-nf_2 (where $n=1$ for normal mixing with two nearly equal frequencies, and $n=2$, 3, 4 for harmonic mixing) can generate its own harmonics in the diode. Since this frequency is low the conversion efficiency is much higher than for microwave frequencies, and on the oscilloscope it cannot be distinguished from the higher frequency harmonic beats. Instead of saying the frequency nf_1-mnf_2 is generated, we state that the nth of f_1-mf_2 is generated in the diode.

Only the interesting experiment with the OCS gas absorption cell shows clearly the existence of harmonics up to the sixth.

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[1] R. G. Strauch, R. A. Miesch, and R. E. Cupp, *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-13, p. 873, November 1965.
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[3] ——, "Heterodyne beats between submillimeter wave components generated in a crystal detector," *Mem. Fac. Engrg. Osaka (Japan) City Univ.*, vol. 6, pp. 93–106, December 1964.

Authors' Reply[1]

Schulten and Stoll are correct when they point out that harmonics of the fundamental beat frequency can produce the IF beats detected. However our experiments show the beats are observed independent of this low-frequency harmonic generation. For example, a signal of comparable magnitude is detected when a 70 GHz signal is mixed with a 59.503 GHz signal using a 60 MHz IF amplifier. In this case, as pointed out by Frenkel,[2] there are also numerous possibilities for producing the beat other than the 17th harmonic of 70 mixing with the 20th harmonic of 59.503. One of these is the 20th harmonic of the 10.497 GHz fundamental beat mixing with the 3rd harmonic of 70 GHz. Whether these possible combinations are more likely than the high frequency harmonic mixing is not readily apparent.

If the fundamental beat occurs at microwave frequencies it is difficult to prevent these currents from producing harmonics, but if the low-frequency currents described by Schulten and Stoll are prevented from entering the external circuit, these currents will be small since they must pass through the millimeter wave by-pass capacitance. By noting the effects on the output as the impedance for these low-frequency currents is controlled, we conclude that the harmonic beats are produced by microwave or millimeter wave harmonic generation and mixing.

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[1] Manuscript received July 12, 1966.
[2] Private communication.

Messrs. Schulten and Stoll[3]

There may be a possibility of distinguishing between beats of the high-frequency harmonics and the harmonics of the fundamental beat frequency by, for instance, controlling the low-frequency impedance, as mentioned in the reply, but as long as the microwave harmonics are not radiated and detected separately the proof of their existence will be dubious.

[3] Manuscript received September 1, 1966.

Propagation in Cylindrical Waveguide Containing Inhomogeneous Dielectric

Considerable attention is being paid to the problem of propagation of electromagnetic waves in inhomogeneous media. Little appears to have been published in systems with cylindrical symmetry. Typical of a numerical approach is the paper by Clarricoats and Oliner[1] where an equivalent circuit is used to represent the inhomogeneous waveguide. On the other hand, Yamada and Watanabe[2] have solved analytically a special case where the dielectric constant is a parabolic function of the radius. The solution obtained was restricted to circularly symmetrical modes, and was exact for TE modes only.

This note deals with another special case where the dielectric constant varies inversely as the square of the radius. Exact analytic solutions are obtained for all possible modes, and conditions for the existence of these modes are established.

As far as we were able to ascertain this is the only analytic solution for both TE and TM type modes in inhomogeneous media in cylindrical coordinates which is known to date. As such, it may serve as a guide in looking for other cases amenable to analytic approach. However, the problem solved is a very special

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[1] P. J. B. Clarricoats and A. A. Oliner, "Transverse-network representation for inhomogeneously filled circular waveguides," *Proc. IEE (London)*, vol. 112, pp. 883–894, May 1965.
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case and may not be representative of other cases.

The wave equations for E and H pertaining to inhomogeneous media are:

$$\nabla \times \nabla \times E - k_0^2 \epsilon_r E = 0 \quad (1)$$

$$\nabla \times \nabla \times H - \nabla \ln \epsilon_r \times \nabla \times H - k_0^2 \epsilon_r H = 0 \quad (2)$$

where E and H are the electric and magnetic field components, $k_0^2 = \omega^2 \epsilon_0 \mu_0$ and ϵ_r is the relative dielectric constant, assumed a function of the radial distance only.

As is usual, the following functional forms are assumed for E and H:

$$E_r = f_1(r) \cos n\phi \quad H_r = g_1(r) \sin n\phi$$
$$E_\phi = f_2(r) \sin n\phi \quad H_\phi = g_2(r) \cos n\phi$$
$$E_z = f_3(r) \cos n\phi \quad H_3 = g_3(r) \sin n\phi. \quad (3)$$

A factor $\exp j(\omega t - k_z z)$ is suppressed. In terms of f and g, (1) and (2) reduce to the following set of coupled equations:

$$\frac{\partial^2 f_1}{\partial r^2} + \left(\frac{1}{r} + \frac{\partial \ln \epsilon_r}{\partial r}\right)\frac{\partial f_1}{\partial r} + \left(k_0^2 \epsilon_r + \frac{\partial^2 \ln \epsilon_r}{\partial r^2} - \frac{1+n^2}{r^2} - k_z^2\right) f_1 = \frac{2n}{r^2} f_2 \quad (4)$$

$$\frac{\partial^2 f_2}{\partial r^2} + \frac{1}{r}\frac{\partial f_2}{\partial r} + \left(k_0^2 \epsilon_r - \frac{1+n^2}{r^2} - k_z^2\right) f_2 = n\left(\frac{2}{r^2} + \frac{1}{r}\frac{\partial \ln \epsilon_r}{\partial r}\right) f_1 \quad (5)$$

$$\frac{\partial^2 f_3}{\partial r^2} + \frac{1}{r}\frac{\partial f_3}{\partial r} + \left(k_0^2 \epsilon_r - \frac{n^2}{r^2} - k_z^2\right) f_3 = jk_z \frac{\partial \ln \epsilon_r}{\partial r} f_1 \quad (6)$$

$$\frac{\partial^2 g_1}{\partial r^2} + \frac{1}{r}\frac{\partial g_1}{\partial r} + \left(k_0^2 \epsilon_r - \frac{1+n^2}{r^2} - k_z^2\right) g_1 = -\frac{2n}{r^2} g_2 \quad (7)$$

$$\frac{\partial^2 g_2}{\partial r^2} + \left(\frac{1}{r} - \frac{\partial \ln \epsilon_r}{\partial r}\right)\frac{\partial g_2}{\partial r} + \left(k_0^2 \epsilon_r - \frac{1+n^2}{r^2} - k_z^2 - \frac{1}{r}\frac{\partial \ln \epsilon_r}{\partial r}\right) g_2 = -n\left(\frac{2}{r^2} + \frac{1}{r}\frac{\partial \ln \epsilon_r}{\partial r}\right) g_1 \quad (8)$$

$$\frac{\partial^2 g_3}{\partial r^2} + \left(\frac{1}{r} - \frac{\partial \ln \epsilon_r}{\partial r}\right)\frac{\partial g_3}{\partial r} + \left(k_0^2 \epsilon_r - \frac{n^2}{r^2} - k_z^2\right) g_3 = jk_z \frac{\partial \ln \epsilon_r}{\partial r} g_1. \quad (9)$$

These equations become uncoupled when

$$\text{a)}\ \frac{\partial}{\partial \phi} = 0; \quad \text{or} \quad n = 0;$$

$$\text{b)}\ \epsilon_r = \frac{l}{r^2} \quad (10)$$

where l is a constant. We consider case b when (5) reduces to:

$$\frac{\partial^2 f_2}{\partial r^2} + \frac{1}{r}\frac{\partial f_2}{\partial r} + \left(\frac{k_0^2 l - 1 - n^2}{r^2} - k_z^2\right) f_2 = 0. \quad (11)$$

Its solution is

$$f_2(r) = A I_m(k_z r) + B K_m(k_z r) \quad (12)$$

where I_m and K_m are the modified Bessel functions of order m and $m^2 = 1 + n^2 - k_0^2 l$. A and B are constants. Their ratio is determined by the boundary conditions $f_2 = 0$ at $r = a, b$, say:

$$-\frac{B}{A} = \frac{I_m(k_z a)}{K_m(k_z a)} = \frac{I_m(k_z b)}{K_m(k_z b)}. \quad (13)$$

This equation in k_z has solutions for imaginary m only, hence the cutoff condition:

$$k_z^2 > \frac{1+n^2}{\quad}. \quad (14)$$

The equation determines the possible values of k_z also. Similarly (8) reduces to:

$$\frac{\partial^2 g_2}{\partial r^2} + \frac{3}{r}\frac{\partial g_2}{\partial r} + \left(\frac{k_0^2 l + 1 - n^2}{r^2} - k_z^2\right) g_2 = 0. \quad (15)$$

Its solution is:

$$g_2(r) = \frac{1}{r}\left[A' I_p(k_z r) + B' K_p(k_z r)\right] \quad (16)$$

where A' and B' are constants, and $p^2 = n^2 - k_0^2 l$.

All other field components can be determined in terms of $f_2(r)$ and $g_2(r)$, for example

$$f_3(r) = -\frac{j}{\alpha}\left[r^2 \frac{\partial g_2}{\partial r} + r g_2 + r\beta f_2\right]$$

$$\alpha = \frac{k_0^2 l - n^2}{\omega\mu}; \quad \beta = \frac{n k_z}{\omega\mu}. \quad (17)$$

On applying boundary conditions to f_3 we find

$$-\frac{B'}{A'} = \frac{I_p'(k_z a)}{K_p'(k_z a)} = \frac{I_p'(k_z b)}{K_p'(k_z b)} \quad (18)$$

where I' and K' are the derivatives with respect to r.

It will be observed that (13) and (18) represent different conditions on k_z. If both these equations are not satisfied by the same value of k_z, the solution must be regarded as a TE wave with finite $f_2\ g_1\ g_3$ and a TM wave with $g_2\ f_1\ f_3$. These solutions do not satisfy the divergence equation unless $n=0$. They represent different modes with different propagation constants.

For some values of the parameters a, b, l, (13) and (18) are satisfied simultaneously. In this special case solutions exist for $n \neq 0$ also. The divergence equation $\nabla \cdot D = 0$ serving to reduce the number of arbitrary constants $A\ B\ A'\ B'$ to one. The solution represents EH type modes.

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Dielectric Constant of Atlas at 9363 MHz

Monoarseniate double d'ammonium de thallium (Atlas), $(NH_4)_4Tl_3(H_2AsO_4)_7$, has been reported to be ferroelectric. The complex dielectric constant of Atlas has been measured by LeDonche [1]. These measurements made at 50 Hz displayed ferroelectric hysteresis loops in the temperature range between 150°K and 110°K with maximum dielectric constant occurring at 150°K. At room temperature Atlas is hexagonal.[1] The present correspondence reports measurements of the temperature dependence of the dielectric constant at 9363 MHz at right angles to the "c" axis of the crystal, i.e., along the length of the crystal.

The complex dielectric constant was measured by the cavity perturbation [2] method. For a rod placed parallel to, and at the maximum value of the electric field, one can write [2]

$$\left(\frac{\delta\omega}{\omega}\right) - j\delta\left(\frac{1}{2Q}\right) = (\epsilon' - 1 - j\epsilon'')\int_{\Delta V} E^2 dv / 2\int_V E^2 dv \quad (1)$$

where ω is the angular resonant frequency, Q is the quality factor, and V is the volume of the cavity. E is the field inside the empty cavity and is assumed to be unaffected by the introduction of the specimens of volume ΔV. The variations $\delta\omega$ and $\delta(1/2Q)$ represent shifts due to the insertion of the specimen. The real part of the relative dielectric constant is designated ϵ', the imaginary part ϵ''. Similar expressions are given by Champlin and Krongard [3].

For the dielectric material placed at the center of a rectangular waveguide resonating in the TE_{10n} mode one has [4]

$$\left(\frac{\delta\omega}{\omega}\right) = -(\epsilon' - 1)\frac{2\Delta V}{V} \quad (2)$$

and

$$\frac{1}{Q_e} = \epsilon'' \frac{4\Delta Y}{V} \quad (3)$$

$$\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_u} \quad (4)$$

where Q_e, Q_u, and Q_L are the quality factors of the dielectric material, the unloaded and the loaded cavity, respectively.

The essential parts of the experimental set-up are shown in Fig. 1. The klystron was swept by the sawtooth voltage through the cavity resonance which was displayed on CRO 2. The klystron output, over the mode, was displayed on CRO 1. The time sweep of both the oscilloscopes were carefully adjusted to be the same. The horizontal scale of CRO 1 was calibrated in frequency with the wavemeter and this calibration was used to measure the 3 dB points of the cavity resonance on CRO 2.

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